Lagrangian Relaxation

- A Lagrangian relaxation is obtained by relaxing a set of constraints from the original formulation to improve tractability.
- However, we also try to improve the bound by modifying the objective function, penalizing violation of the dropped constraints.
- Consider a pure IP defined by

$$\begin{aligned} \max\ & c^\top x \\ \text{s.t. } & A'x \le b' \\ & A''x \le b'' \\ & x \in \mathbb{Z}^n_+, \end{aligned} \qquad \text{(IP)}$$

where $\mathcal{S}_R = \{x \in \mathbb{Z}^n_+ \mid A'x \le b'\}$ bounded and optimization over S_R is "easy."

- Lagrangian Relaxation:

$$LR(u) : z_{LR}(u) = ub'' + \max_{x \in \mathcal{S}_R} \{(c - uA'')x\}.$$